Chapter 12 – Periodic Motion

Introduction

Read the Dominion Modeling Problem on “Mach Speed” on page 265.

• What limits the speed of sound in air?
• What happens when an object (plane, bullet, rocket) exceeds the speed of sound?
• What does the “Mach” speed number mean?

Section 12A – Simple Harmonic Motion

Periodic Motion is motion that repeats at a constant rate – e.g., a plucked violin string

• For a mass at the end of a spring, the mass will oscillate up and down repetitively
• For a mass at the end of a pendulum, the mass will oscillate back and forth repetitively

Simple harmonic motion (SHM) is motion controlled by a restoring force that is proportional to the displacement of the mass from its equilibrium position

• A mass at the end of a spring following Hooke’s law $F = -k\Delta x$
• A mass at the end of a pendulum at small angles $F = -mg\sin\theta$

Vibrations, which represent simple harmonic motion, exist all around us. Some examples include:

• Vibrating air carries sound to our ears
• Vibrating electromagnetic fields carry light to our eyes
• Vibrating atoms carry heat to our fingertips
• Vibrating radio waves allows us to communication wirelessly
• Vibrating microwaves speed cooking
• Vibrating crystals help electronic watches tell time
• Vibrating engine pistons help move cars

Some of the common properties for simple harmonic motion include:

• Cycle is the complete path from one position to that same position on its next pass (unitless).
• Amplitude is the maximum displacement from the equilibrium position in meters (m).
• Frequency is the number of complete cycles per second, in hertz (Hz) where 1 Hz = 1 s⁻¹
• Period is the time in seconds required to complete one cycle in seconds (s)

Restoring Force is the sum of forces attempting to bring a mass back into equilibrium

• Restoring force is always negative because it tends to move the object back toward equilibrium
• For a spring, the restoring force is due to Hooke’s Law for the stretched or compressed spring
• For a pendulum, the restoring force is due to the force of gravity for the hanging mass

Table 12-1 compares position, restoring force, acceleration, and velocity for an oscillating spring.
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Reference Circle

- Circular motion (in two dimensions) is like simple harmonic motion
- The profile (or shadow) of a mass in circular motion viewed from the side matches the profile of:
  - A mass on a spring viewed from the side
  - A mass on a pendulum viewed from the top
- An object’s reference circle can help you visualize its harmonic motion.
  - The period of a circle is \( T = \frac{2\pi r}{v_t} \), where \( r \) is the radius and \( v_t \) is the tangential velocity
  - Since, for a mass on a spring, \( \frac{1}{2} m v_t^2 = \frac{1}{2} k\Delta x^2 \), then its velocity is \( v_t = \Delta x \sqrt{\frac{k}{m}} \),
  - Equating the radius of the reference circle, \( r \), to the spring displacement, \( \Delta x \)
    \[ T = \frac{2\pi}{v_t} = \frac{2\pi \Delta x}{v_t} = \frac{2\pi \Delta x}{\Delta x} \sqrt{\frac{m}{k}} \]
    \[ T = 2\pi \sqrt{\frac{m}{k}} \]  
    Equation 12.6, page 268
  - The period of a mass at the end of a spring is independent of the actual displacement
- A “more proper” solution requires calculus using Newton’s second law and recognizes that:
  - The restoring force is proportional to displacement: \( F = -kx \)
  - The acceleration is the second derivative of displacement: \( F = ma = m \frac{d^2x}{dt^2} \)
    \[ m \frac{d^2x}{dt^2} = -kx \rightarrow x = A \sin(\omega t) \), where \( A \) = amplitude, and \( \omega = \sqrt{\frac{k}{m}} \)
  - And, since \( T = \frac{2\pi}{\omega} \), then \( T = 2\pi \sqrt{\frac{m}{k}} \)

Work section 12A review problem 5 on page 269.
Work section 12A review problem 6 on page 269.
Work section 12A review problem 7 on page 269.
Work section 12A review problem 8 on page 269.

Section 12B – Periodic Motion and the Pendulum

Historical Overview

- Aristotle and the early Greeks could not explain continuous back-and-forth motion of pendulums because they thought an object should continue in a single direction until it was stopped
- Galileo observed that the period of oscillating chandeliers was independent of their amplitude
  - Pendulums with small displacement oscillate at the same frequency (or period) as those with large displacement

An Ideal Pendulum consists of a mass hanging at the end of an ideal string or massless rod

- The distance from the pivot point to the center of mass, \( L \), equals the length of the string or rod
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Two forces act on an ideal (or any) freely moving pendulum

• The downward weight of the hanging mass
• The upward tension of the string or rod (pivot arm)

The motion of a pendulum can only be tangential because the string or rod constrains its motion.

• The only tangential force on the system is the tangential component of the weight
  \( F_T = |mg| \sin \theta \)
• The radial forces acting on the system are the radial component of the weight and the centripetal force of the moving mass
  \( F_R = |mg| \cos \theta \)
  \( F_C = m \frac{v_T^2}{L} \)

In general, the resulting equation of motion is difficult to solve except for small angles. But at small angles, we can approximate: \( \sin \theta \approx \theta \)

• For \( \theta = 0.1 \) (about 5.73°), \( \sin \theta = 0.09983 \) (< 0.17% error)
• For \( \theta = 0.2 \) (about 11.46°), \( \sin \theta = 0.19867 \) (< 0.67% error)
• The error for \( \theta = 0.25 \) (about 14.32°) is only about 1.04%

Using the reference circle and small angle approximation, we can solve for the pendulum’s period.

• The tangential restoring force of the pendulum is \( F = -mg \sin \theta \),
• For small angles, \( \theta \approx \frac{x}{L} \), where \( x \) is the displacement and \( L \) is the pendulum’s length
• \( F \approx -\frac{mg}{L} x \) which also has the form \( F = -kx \) (which we’ve solved above) where \( k = \frac{mg}{L} \)
• If we substitute this value of \( k \) back into equation 12.6, we have \( T = \frac{2\pi}{\sqrt{\frac{mg}{mL}}} \)
• \( T = 2\pi \sqrt{\frac{L}{g}} \)  \hspace{1cm} \text{Equation 12.9, page 273}
• The period for small angles is independent of both mass and displacement

Unlike ideal pendulums, real pendulums have both mass and shape

• We must utilize the moment of inertia, \( I \), of the pendulum (chapter 11)
  \( \circ \) For the simple, ideal pendulum as a mass at the end of a string, \( I = mL^2 \)
  \( \circ \) For a thin rod having a length \( L \) and mass \( m \), \( I = \frac{mL^2}{3} \) (Appendix F, page 699)
• We also must utilize the center of mass, \( l \), of the pendulum (chapter 11)
  \( \circ \) For a thin rod, the center of mass is \( l = \frac{L}{2} \)
• Experimentally, the period of a real pendulum is, \( T = \frac{2\pi}{\sqrt{\frac{mg}{ml^2}}} \), or
• \( T = 2\pi \sqrt{\frac{L}{mg}} \)  \hspace{1cm} \text{Equation 12.10, page 275}
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The Foucault Pendulum

- The earth’s rotation is not a force, so it does not directly act upon a pendulum’s motion
  - What really happens is that the earth rotates beneath the swinging pendulum
  - The pendulum’s back-and-forth path will appear to rotate in a circle over time
- Jean Foucault (France, 1819-1868) demonstrated this in 1851 using a 28-kg mass attached to a 67.5 m wire hanging from the top of the Pantheon in Paris, France
- The period of the rotation is different at different latitudes on the earth
  - At the equator, the pendulum does not rotate (the period is infinite)
  - At either pole, the pendulum will rotate once per day (the period is 23.93 h)
  - At latitudes between, the period is \( T = \frac{23.93 \text{ hr}}{\sin \theta} \) where \( \theta \) is the latitude
    - For Jacksonville, at about 30° latitude, the period is about 2 days (48 h)

Section 12C – Oscillations in the Real World

Damping

- A Damped Harmonic Oscillation is a vibrating oscillator under the influence of frictional forces
- Resistance eventually causes the amplitude of an oscillation to decrease and finally stop
  - Resistance produces an opposing force proportional to velocity (Equation 12.11)
- There are generally three types of damping:
  - Underdamping – completes multiple cycles with decreasing amplitude (Figure 12-16)
  - Critical damping – completes one cycle with minimal overshoot (Figure 12-18)
  - Overdamping – does not even complete one single cycle (Figure 12-17)
- Automobile “struts” try to achieve a condition of critical damping for a more comfortable ride

Driven Oscillations and Resonance

- Resonant Frequency is the frequency of oscillation that an object will have when it is left alone
  - This may also be known as the “harmonic frequency” or “natural frequency”
  - This is like the frequency of a spring or pendulum from sections 12A and 12B
- Driven Oscillation causes damped oscillators to continue due to an applied external force.
  - This is like your pushing a friend in a swing to keep him or her swinging
- Resonant Oscillation occurs when the driving force is applied at the same frequency as the resonant frequency
  - If you drive an object faster or slower than the resonant frequency, the force will be out of phase with the motion and the amplitude will be less than the natural amplitude
  - If you drive an object at the resonant frequency, the amplitude will increase and only the natural damping forces will keep it from breaking up (see figure 12-21)
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- If the driving force continues to exceed the damping force, the object may be destroyed (see the pictures of destruction of the Tacoma Narrows Bridge in figure 12-22)

Discuss section 12C review problem 7 on page 280.

Section 12D – Waves

Waves

- **Waves** are oscillations of extended bodies by creating a periodic “disturbance” (or motion) though an extended “medium” (or material)
  - Examples include: ocean waves, sound waves, or waves in a vibrating string
  - The wave moves in some direction while the medium itself vibrates back-and-forth
  - Energy is transmitted by the periodic conversion between kinetic and potential energy
- **Transverse Waves** cause the medium to vibrate perpendicular to the direction of travel
  - Rope waves are transverse (see Figure 12-23)
  - Light and radio waves are transverse, but electromagnetic waves don’t require a medium
- **Longitudinal Waves** cause the medium to vibrate in the direction of travel
  - Sound waves are longitudinal (see Figure 12-24)
  - **Compression** occurs when the particles of the medium are closer together
  - **Rarefaction** occurs when the particles of the medium are farther apart
- Waves are described by three measurable properties:
  - **Velocity** (v) – the speed of the disturbance through the medium
    - The speed of sound in dry air is about 343 m/s
    - The speed of electromagnetic waves in vacuum is $3.00 \times 10^8$ m/s
  - **Frequency** (f) – the number of cycles of the disturbance per second
    - The sound wave made by middle C is 261.6 Hz
    - The light wave that appears red to the eye is approximately $4.3 \times 10^{14}$ Hz
  - **Wavelength** ($\lambda$) – the distance between corresponding “crests” or “troughs” in one cycle
    - Middle C has a wavelength of about 1.31 m
    - Red light has a wavelength of about $697 \times 10^{-9}$ m (697 nm)
- These three properties are related by the relatively simple equation
  - $v = \lambda f$  
  
Equation 12.12

Work section 12D review problem 7 on page 286.
Work section 12D review problem 8 on page 286.

Sound Waves

- Sound waves are longitudinal waves that come from a vibrating body detected by your ears
- Sound travels by compression and rarefaction, so it must have a medium to travel through
  - Light and radio waves do not require a medium and can travel in a vacuum
  - Sound waves cannot travel in a vacuum
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- Properties of sound waves that are important to music are:
  - **Intensity** \((W/m^2)\) determines the “loudness” of sound
    - The human ear is “logarithmic” in detecting of sound loudness
    - A sound must have 10 times the intensity to “sound” twice as loud
    - Loudness is measured in “decibels” \((db)\) as
      \[
      \beta = (10db) \log \left( \frac{I}{10^{-12} W/m^2} \right)
      \]
    - The threshold of human hearing varies with frequency (see Figure 12-28)
  - **Pitch** is equivalent to frequency
    - High pitch has high frequency
    - Low pitch has low frequency
  - **Quality** defines how frequencies combine to cause different instruments to sound unique
    - High C on a trumpet sounds much different from that on a violin or piano
    - **Fundamental Frequency** is the dominant frequency
    - **Harmonic Frequencies** are multiples of the fundamental frequency

The **Doppler Effect** is the property of a sound to have a different pitch than natural while moving (see Figure 12-29)

- Pitch is higher when the sound emitting object is moving toward you
- Pitch is lower when the sound emitting object is moving away from you
- Think back when you heard the siren of a passing fire truck or police car
  - The pitch is higher in the approaching siren because the speed of the fire truck makes the sound waves closer together than they would be if it were standing still
  - The pitch is lower in the receding siren because the speed of the fire truck makes the sound waves farther apart than they would be if it were standing still
- The Doppler effect manifests itself in light waves as a “blue shift” (higher frequencies) when moving toward you or as a “red shift” (lower frequencies) when moving away from you

**Work section 12D review problem 9 on page 286.**

Re-read the Dominion Modeling Problem on “Mach Speed” on page 265.

- A substance cannot vibrate faster than the speed of sound in that substance
  - The speed of sound in air under “average” conditions is about \(343 m/s\) \((1,125 ft/s)\)
  - Denser materials have higher speeds of sound \((water \approx 1,480 m/s; steel \approx 5,930 m/s)\)
- When an object travels **near** the speed of sound, several things happen
  - The air molecules are compressed near the front of the object creating a “shock” cone
  - The air molecules are rarefied behind the object as they are “outrun” by the object
- When an object travels **faster** than the speed of sound, then the shock wave is heard as a sonic “boom” on the ground
- Objects traveling at the speed of sound are traveling at “Mach 1” – a useful, non-exact quantity because the “local” speed of sound in air varies